AD-A076 494 NAVAL UNDERWATER SYSTEMS CENTER NEW LONDON CT NEW LO--ETC F/G 17/1
EFFECTS OF RANDOM SHADING, PHASING ERRORS, AND ELEMENT FAILURES--ETC(U)
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NUSC Technical Document 6103



# Effects of Random Shading, Phasing Errors, And Element Failures On the Beam Patterns Of Line and Planar Arrays

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17 September 1979



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## Preface

This document was prepared under NUSC Project No. A12212, "RAPLOC/Wide Aperture Array," Principal Investigators, J. Bradshaw (Code 325) and A. Quazi (Code 313), Program Manager, J. Hall (SEA-3211), Project Manager, Capt. W. White (SEA-6343), Sponsor, Capt. J. J. King (OP-224F).

This document was originally presented at ICASSP 79 International Conference on Acoustics, Speech, and Signal Processing at Washington, DC, on 2 to 4 April 1979.

The derivations of all equations and corresponding programs in this document will be presented in a technical report by A. H. Nuttall (Code 313), New London Laboratory, Naval Underwater Systems Center, in the near future.

**Reviewed and Approved:** 17 September 1979

**R. W. Hasse**
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New London, Connecticut 06320.

(9) Technical document,

## REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

**1. REPORT NUMBER**
TD 6103

**2. GOVT ACCESSION NO.**

**3. RECIPIENT'S CATALOG NUMBER**

**4. TITLE (and Subtitle)**
(6) EFFECTS OF RANDOM SHADING, PHASING ERRORS, AND ELEMENT FAILURES ON THE BEAM PATTERNS OF LINE AND PLANAR ARRAYS.

**5. TYPE OF REPORT & PERIOD COVERED**

**6. PERFORMING ORG. REPORT NUMBER**

**7. AUTHOR(s)**
(10) Azizul H. Quazi
Albert H. Nuttall

**8. CONTRACT OR GRANT NUMBER(s)**

**9. PERFORMING ORGANIZATION NAME AND ADDRESS**
Naval Underwater Systems Center
New London Laboratory
New London, CT 06320

**10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS**
A12212

**11. CONTROLLING OFFICE NAME AND ADDRESS**
Naval Sea Systems Command (SEA-6343)
Washington, DC 20360

**12. REPORT DATE**
(11) 17 Sep[redacted]79

**13. NUMBER OF PAGES**
10

**14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)**
(12) 14

**15. SECURITY CLASS. (of this report)**
UNCLASSIFIED

**15a. DECLASSIFICATION/DOWNGRADING SCHEDULE**

**16. DISTRIBUTION STATEMENT (of this Report)**
Approved for public release; distribution unlimited.
(14) NUSC-TD-6103

**17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)**

**18. SUPPLEMENTARY NOTES**

**19. KEY WORDS (Continue on reverse side if necessary and identify by block number)**

| | |
|---|---|
| Beam Patterns | Planar Arrays |
| Element Failure | Random Phasing Errors |
| Line Arrays | Random Shading |
| Phasing Errors | Side Lobe Level |

**20. ABSTRACT (Continue on reverse side if necessary and identify by block number)**
The effects of random shading, phasing errors, and element failures on the beam patterns of line and planar arrays are investigated. The results indicate that the effects of element failures and random phasing errors on the beam pattern are more pronounced than the effect of shading errors. The deep side lobe level, in both line and planar arrays, is critically affected by random errors that include shading, phasing, and element failures. The combined effect of random shading, phase, and element failures on the beam pattern of a line array is more dramatic than it is on the beam pattern of a planar array.

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# EFFECTS OF RANDOM SHADING, PHASING ERRORS, AND ELEMENT FAILURES ON THE BEAM PATTERNS OF LINE AND PLANAR ARRAYS

## INTRODUCTION

The hydrophone signals and noises that are received by an array pass through an analog channel that provides amplification, filtering, and analog-to-digital (A/D) conversion. The outputs are inserted into a digital beamformer, where they are appropriately shifted in phase (delayed) and summed to form beams. When shading is accomplished in the beamformer, the signals will also be subjected to digital attenuation that depends on the element position in the array. When each signal passes through the entire channel, including digital shading (attenuation), the signal is subjected to an overall channel gain and phase shift that is frequency dependent. If all channels have identical gain and identical phase shift, and if all digital delays are exact for the desired beam, the summed signal will represent perfectly the phase of the acoustic plane-wave signal at the array.

In practice, each signal channel in the array will have imperfections; therefore, each output is not at its expected amplitude or phase. Thus, an array designed for a desired beam pattern will have a beam pattern different from the expected results. Random shading errors and phase errors will affect the beam pattern, directivity index, and beam-pointing direction. The shading errors are introduced through errors in the weighting, variations in the transducers' sensitivities, quantification of the shading coefficients of the elements, and amplifier-gain errors in the system. Phase errors are introduced by errors in the placement of the array elements, frequency characteristics of the filter that is included in the signal channel, and digital phase shifts or time delays. The most severe type of shading error is the failure of an element, which corresponds to a shading coefficient of zero.

Summary results of an investigation of the effects of random shading errors, phasing errors, and element failures on the beam patterns (especially the side lobe levels) of line and planar arrays are presented in this document.

## BEAM PATTERNS

The amplitude beam pat... of a planar array of MXN elements arranged in a rectangular grid in the xy plane, with spacings $d_x$ and $d_y$ between elements, can be written as

$$u(\theta,\phi) = \sum_{m=1}^{M} \sum_{n=1}^{N} W'_{mn} \exp\left[j\frac{2\pi}{\lambda} \sin\theta(m \cdot d_x \cdot \cos\phi + n \cdot d_y \cdot \sin\phi)\right] , \quad (1)$$

where $W'_{mn}$ is the actual shading coefficient at the mn-th element of the array, $\theta$ and $\phi$ are the (polar and azimuthal) angular coordinates, and $\lambda$ is the wavelength of the signal.

The (power) beam pattern is obtained by multiplying equation (1) by its complex conjugate. The factor $W'_{mn}$ is the actual shading coefficient at the mn-th element and is related to the designed (error-free) shading coefficient $W_{mn}$ in the following manner

$$W'_{mn} = W_{mn}\alpha_{mn}(1 + \Delta_{mn}) \exp(j\delta_{mn}) \ , \tag{2}$$

where $\Delta_{mn}$ is the fractional error in the weight and $\delta_{mn}$ is the error in the phase. The average values of the phase error and weight error are assumed to be zero. The phase and amplitude errors at any element are taken to be independent of the errors at any other element. The factor $\alpha_{mn}$ accounts for missing elements, such as might be caused by element failure. It has the value unity with probability Pe and the value zero with probability (1 - Pe). Thus, the probability of the element mn being operative is designated Pe; this probability is assumed to be independent of the location of the element within the array. The probability Pe is also equal to the average fractional number of elements that remain operative.

It can be shown that the average power pattern is[1,2,3]

$$\begin{aligned}\overline{|u(\theta,\phi)|^2} &= Pe^2 e^{-\overline{\delta^2}} |u_0(\theta,\phi)|^2 \\ &+ \left[(1 + \overline{\Delta^2})Pe - Pe^2 e^{-\overline{\delta^2}}\right] \sum_{m=1}^{M} \sum_{n=1}^{N} W_{mn}^2 \ ,\end{aligned} \tag{3}$$

where $\overline{\delta^2}$ and $\overline{\Delta^2}$ are the variances of the phase and weight errors, respectively, and $W_{mn}$ is the designed weight. $|u_0(\theta,\phi)|^2$ is the error-free designed normalized beam pattern.

Equation (3) shows that the effect of random phase and weight errors is to produce an average power pattern that is a superposition of two terms. The first term is the error-free power pattern multiplied by the square of the fraction of elements remaining operative, and by a factor proportional to the phase error. The second term depends on both weight and phase errors as well as the average fraction of the elements that are operative. Also, it depends on the exact distribution of the aperture weights, but it is independent of the angular coordinates, $\theta,\phi$. The second term can be thought of as a "statistically omnidirectional" pattern. It causes the deep side lobes of the pattern to differ considerably in the presence of error from those with no error. The shapes of the main lobe and close-in side lobes are relatively unaffected by the random errors. Notice that all the directional properties are in the first term; it is evident that the shape of the average power pattern, as represented by the first term, is unchanged from the error-free power pattern. Hence, the beamwidth remains unchanged. The dominant effect is a reduction in gain as seen by the coefficient of the error-free power pattern.

Beam patterns of an equispaced line array with 32 elements and equal shading coefficients, with a standard deviation of relative weight error of $\sigma_\alpha = 0.06$, are shown in figures 1 through 3 in terms of the fundamentally dimensionless quantity $u = 2\pi d/\lambda(\sin\theta - \sin\theta_0)$, where d is the spacing between elements, $\lambda$ is the wavelength of the incoming signal, $\theta$ is the direction of signal arrival, and $\theta_0$ is the "look" direction (or steered direction). There are four curves (A, B, C, and D) plotted in figure 1. Curve A shows the designed (error-free) power beam pattern. Curve B indicates the actual average power pattern, which includes the effects of random errors. In this figure, only the weight errors are nonzero, with a standard deviation $\sigma_\alpha = 0.06$. Curves C and D show the beam patterns at one and two standard deviations.

The B curve indicates that the effect of shading er[illegible] on the main beam and side lobe level is negligible. However, the effect of random weight error on the one- and two-standard deviation curves in the deep side lobe region is to increase the level about 2 to 4 dB.

In order to see the effects of random phase errors, in addition to weight errors, on the beam pattern of a line array, a phase error standard deviation of $\sigma_\phi = 0.15$ radian is considered. Figure 2 shows the combined effects of weight and phase errors on the beam pattern. The curves indicate that the effects of combined weight and phase errors, especially in the deep side lobe region, are more pronounced (about 5 to 7 dB) than the effect due to weight error alone. Again, there is a main beam loss in gain of about 0.1 dB due to these combined weight and phase errors.

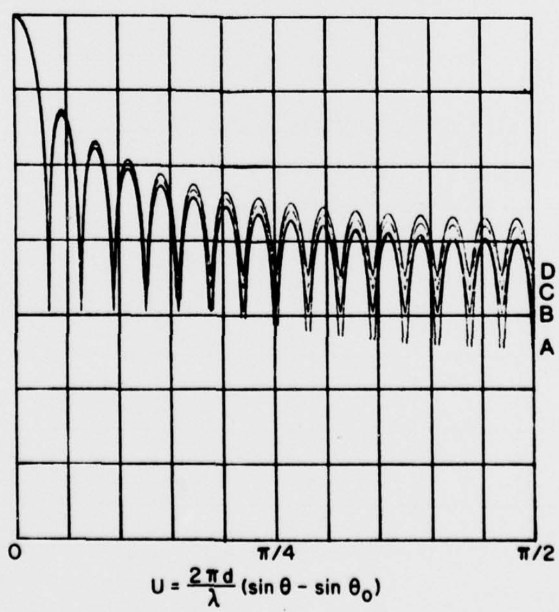


Figure 1. Standard Deviation of Phase Errors, $\sigma_\phi = 0$ Radians, Probability of Element Failure = 0

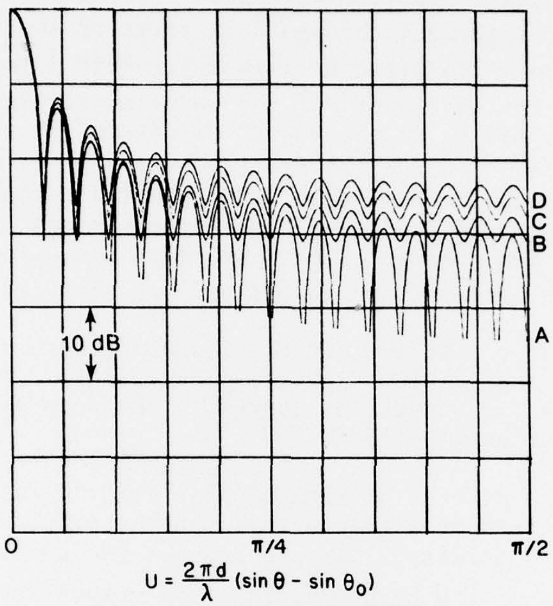


Figure 2. Standard Deviation of Phase Errors, $\sigma_\phi$ = 0.15 Radians, Probability of Element Failure = 0

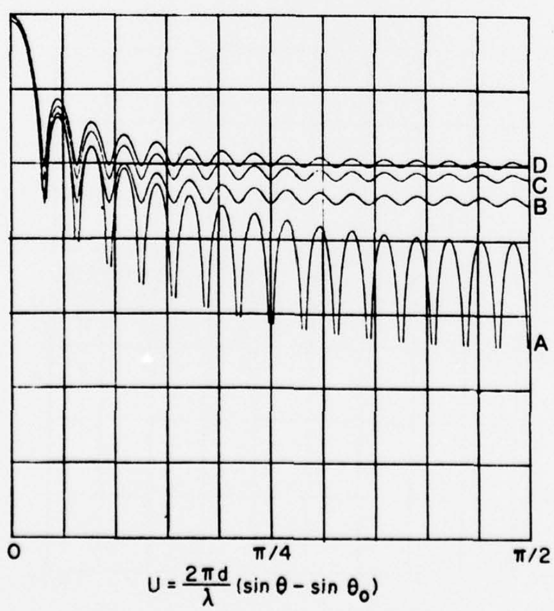


Figure 3. Standard Deviation of Phase Errors, $\sigma_\phi$ = 0.15 Radians, Probability of Element Failure = 0.08

Figure 3 shows the combined effects of weight error, phase error, and element failures. Element failures affect the main beam gain and increase the average side lobe level. It can be seen that the effect of element failures on the side lobe level is dramatic. For example, the combined effect due to weight, phase, and element failures on the deep side lobe level is about 10 dB at the two-sigma level.

Figure 4 shows the beam pattern of a planar array of 32 x 14 elements, spaced at a half wavelength in both the x-direction and y-direction. All weights are equal. The polar and azimuthal look angles are both 0 deg. This beam pattern is a slice through the xz plane from 0 to $\pi/2$ radians in the polar arrival angle. The azimuthal arrival angle of the incoming wave is 0 deg. The effects of random weight error, phase error, and element failure are included in the plotted beam pattern. Curve A shows the ideal designed beam pattern, whereas the B, C, and D curves show the effects of random errors and element failures. Specifically, curves A, C, and D are the average power pattern, the average plus one-sigma pattern, and the average plus two-sigma pattern, respectively. Apparently, the statistically omnidirectional side lobe level, which is independent of the signal arrival angle, is less than that of the designed level. As a result, the shape of the side lobe level (as a function of signal arrival angle) is dominated by the designed side lobe level. In other words, the contribution of the random errors to the side lobe level is small compared to the designed side lobe level. The main-beam loss is about 1 dB, due mainly to element failures.

Figure 5 shows the beam pattern taken on a slice through the yz plane. Here, as in figure 4, the effects of random weight, phase, and element failures are not serious.

Comparing the beam patterns of a line array (figures 1 to 3) with those of a planar array, it is seen that, due to random errors and element failures, the increase in the side lobes is more pronounced in the case of a line array, although all the parameters (such as $\sigma_\alpha$, $\sigma_\phi$, and Pe) have been kept constant. This discrepancy occurs mainly because the planar array has a larger number of elements than the linear array.

Figure 6 shows the beam pattern of a planar array at a polar look angle of $0.25\pi$ radians, an azimuthal look angle of $0.75\pi$ radians, and the azimuthal signal arrival angle of $0.75\pi$ radians. One can easily see that the statistically omnidirectional side lobe level due to random errors and element failures is much higher than the designed side lobe level. As a consequence, the shape and level is dominated in the side lobes by the contribution due to random errors and element failures.

Figure 7 shows an azimuthal slice in arrival angle from 0 to $2\pi$, where the polar look and arrival angles are $0.25\pi$ radians and the azimuthal look angle is $0.75\pi$ radians. In this case (as in figure 6), the contribution due to random errors and element failures is large compared with the designed level. Main beam loss is less than 1 dB.

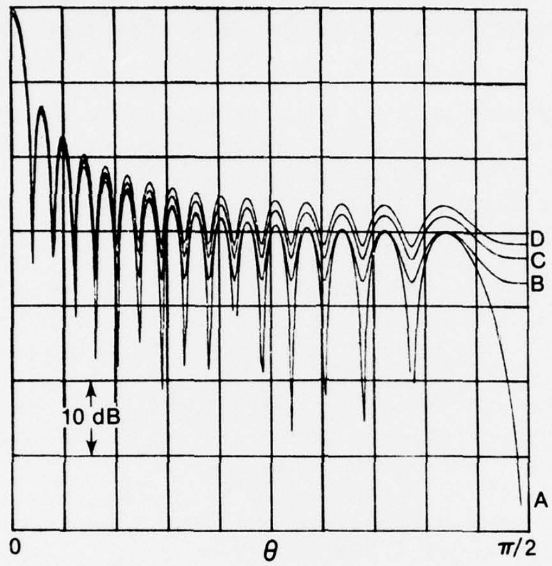


Figure 4. Polar Look Angle = 0 Radians, Azimuthal Look Angle = 0 Radians, Azimuthal Arrival Angle = 0 Radians, and Polar Slice θ = 0.5π Radians

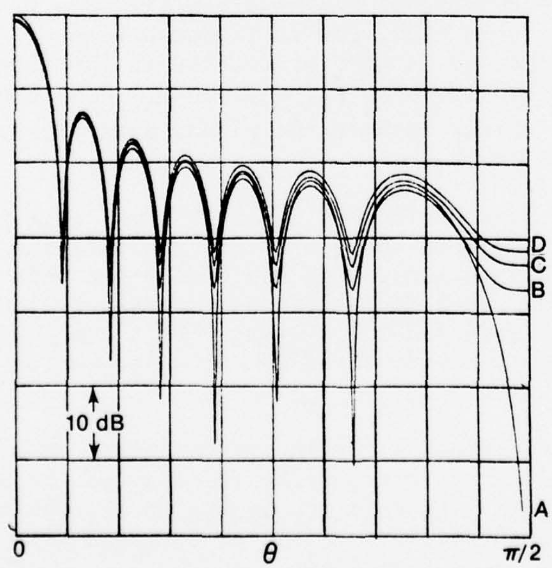


Figure 5. Polar Look Angle = 0 Radians, Azimuthal Look Angle = 0.5π Radians, Azimuthal Arrival Angle = 0.5π Radians, and Polar Slice θ = 0 to 0.5π Radians

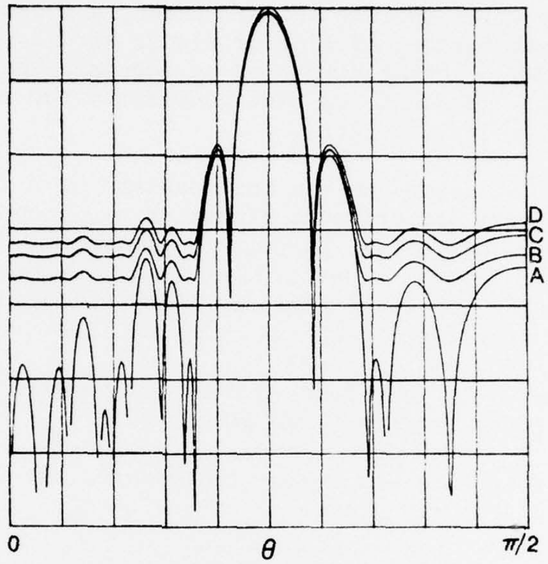


Figure 6. Polar Look Angle = $0.25\pi$ Radians, Azimuthal Look Angle = $0.75\pi$ Radians, Azimuthal Arrival Angle = $0.75\pi$ Radians, and Polar Slice $\theta$ = 0 to $0.5\pi$ Radians

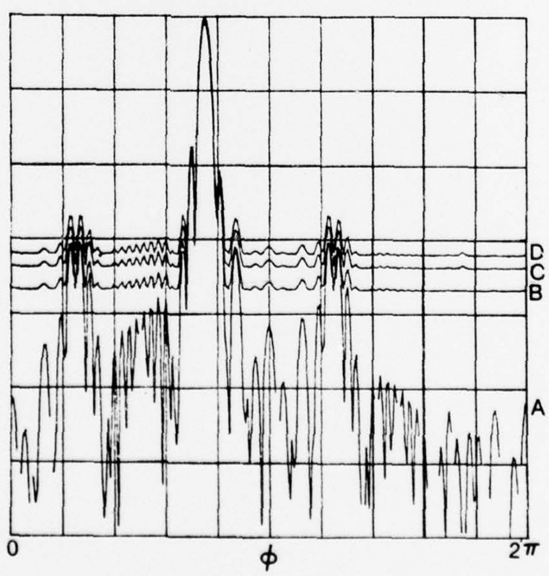


Figure 7. Polar Look Angle = $0.25\pi$ Radians, Azimuthal Look Angle = $0.75\pi$ Radians, Polar Arrival Angle = $0.25\pi$ Radians, and Azimuthal Slice $\phi$ = 0 to $2\pi$ Radians

## SUMMARY AND DISCUSSION

The random errors, which include weight, phase, and element failures in the signal channel in beamforming of line or planar arrays, affect the directivity, width, shape, and pointing direction of the main beam and alter the side lobe level. In this document, we have considered the main beam gain and side lobe level of line and planar arrays.

The effects of random errors on the beam pattern of a line array are more pronounced than on the beam pattern of a planar array, even though all parameters (such as variances of weight, phase, and probability of element failures) are kept constant. This discrepancy exists mainly because the planar array has a significantly larger number of elements (448) compared with the line array (32). The noteworthy effects of random errors are as follows:

1. The rise of the side lobe level due to random errors is relatively independent of the steering angle or look direction.

2. For a given array size and a given tolerance, the increase is most noticeable for the deep side lobe level.

3. Random errors reduce the main beam gain.

4. For a given tolerance and a given designed side lobe level, the rise of the side lobe level would be less for a larger array.

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